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THE NEWSVENDOR PROBLEM

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1. Introduction

Early each morning, the owner of a corner newspaper stand needs to order newspapers for that day. If the owner orders too many newspapers, some papers will have to be thrown away or sold as scrap paper at the end of the day. If the owner does not order enough newspapers, some customers will be disappointed and sales and profit will be lost. The newsvendor problem is to find the best (optimal) number of newspapers to buy that will maximize the expected (average) profit given that the demand distribution and cost parameters are known.

The newsvendor problem is a one-time business decision that occurs in many different business contexts such as:

- **Buying seasonal goods for a retailer** – Retailers have to buy seasonal goods (sometimes called style goods) once per season. (A "season" can be a day, week, year, etc.) For example, most swimsuits can only be purchased seasonally. If a buyer orders too few swimsuits, the retailer will have lost sales and dissatisfied customers. If the buyer orders too many swimsuits, the retailer will have to sell them at a clearance price or even throw some away. Gupta, Hill, and Bouzdine-Chameeva (2006) extend the newsvendor model to handle multiple seasons (periods), each with a different price elasticity of demand.
- **Making the last buy or last production run decision** – Manufacturers have to make a last buy (or last production run) for a product (or component) that is near the end of its life cycle. If the order size is too small, the firm will have stockouts and disappointed customers. If the order size is too large, the firm will only be able to sell the items for their salvage value. Hill, Giard, and Mabert (1989) considered a similar problem within the context of selecting a "keep" quantity for an aging service parts inventory.
- **Setting safety stock levels** – A distributor has to set the safety stock level for an item. If the safety stock is too low, stockouts will occur. If safety stock is too high, the firm has too much carrying cost. Nearly all safety stock models are newsvendor problems with the selling season being one order cycle or one review period.

- **Setting target inventory levels** – A salesperson carries inventory in the trunk of a vehicle. The inventory is controlled by a target inventory level. If the target is too low, stockouts will occur. If the target is too high, the salesperson will have too much carrying cost.
- **Selecting the right capacity for a facility or machine** – If the capacity of a factory or a machine over the planning horizon is set too low, stockouts will occur. If capacity is set too high, the capital costs will be too high.
- **Overbooking customers** – If an airline overbooks too many passengers, it incurs the cost of giving away free tickets to inconvenienced passengers. If the airline does not overbook enough seats, it incurs an opportunity cost of lost revenue from flying with empty seats.

All of these newsvendor problem contexts share a common mathematical structure with the following four elements:

- **A decision variable (Q)** – The newsvendor problem is to find the optimal Q for a one-time decision, where Q is the decision quantity (order quantity, safety stock level, overbooking level, etc.). Q^* denotes the optimal (best) value for Q.
- **Uncertain demand (D)** – Demand is a random variable defined by the demand distribution (e.g., normal distribution, Poisson distribution, etc.) and estimates of the distribution parameters (e.g., mean, standard deviation). Demand may be either discrete (integer) or continuous. This paper develops the newsvendor models for both cases.
- **Unit overage cost (c_o)** – This is the cost of buying one unit more than the demand during the one-period selling season. In the standard retail context, the overage cost is the unit cost (c) less the unit salvage value (s), i.e., $c_o = c - s$. The salvage value is the salvage revenue less the salvage cost required to dispose of the unsold product.
- **Unit underage cost (c_u)** – This is the cost of buying one unit less than the demand during the one-period selling season. This is also known as the stockout (or shortage) cost. In the retail context, the underage cost is computed as the lost contribution to profit, which is the unit price (p) less the unit cost (c), i.e., $c_u = p - c$. The lost customer goodwill (g) associated with a lost sale can also be included (i.e., $c_u = p - c + g$). However, it is difficult to estimate the g parameter because it is the net present value of all future lost profit from this customer and all other customers affected by this customer's negative reports (negative "word of mouth").

Since c_o and c_u are both cost parameters, taxes should be considered for both or neither. Given that the newsvendor problem is in a single period, cash flows do not need to be discounted.

This paper is intended to give readers both a mathematical and intuitive understanding of the newsvendor model that is used to solve the newsvendor problem. This model is one of the most celebrated models in all of operations management and operations research and has been in the literature for over 100 years (Edgeworth 1888; Arrow, Harris, & Marschak 1951).

This paper presents the newsvendor problem in the standard retail context. The reader is encouraged to explore the Excel workbook Newsvendor Model.xls from Clamshell Beach Press. The remainder of this paper is organized as follows. Sections 2 and 3 present the newsvendor problem with discrete (integer) demand and continuous (non-integer) demand. Section 4

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presents a simple example with graphs for the continuous demand case using both the triangular and normal distribution approaches. Section 5 presents a simple way to estimate the critical ratio that is needed for these two models. Section 6 then discusses behavioral issues related to the newsvendor problem. Section 7 concludes the paper with a summary of the main concepts. Appendices 1 and 2 derive the newsvendor model with discrete and continuous demand. Appendix 3 derives the expected values. Appendix 4 presents the VBA code for the inverse Poisson CDF and Appendix 5 presents the VBA code for the inverse triangular CDF.

2. Newsvendor problem with discrete demand

The model

When demand only takes on integer (whole number) values, it is said to be "discrete."[1] With order quantity Q and specific demand D, the cost for the one-period selling season is:[2]

$$Cost(Q,D)=\begin{cases} c_o(Q-D) & \text{if } D<Q \\ c_u(D-Q) & \text{if } D\geq Q \end{cases} \tag{1}$$

For discrete demand, the demand distribution is defined by the probability mass function[3] $p(D)$. The equation for the expected cost, therefore, is given by:

$$ECost(Q)=\sum_{D=0}^{\infty}p(D)Cost(Q,D)=c_o\sum_{D=0}^{Q-1}p(D)(Q-D)+c_u\sum_{D=Q}^{\infty}p(D)(D-Q) \tag{2}$$

The first term in equation (2) is the expected overage (scrap) cost and the second term is the expected underage (shortage) cost. As shown in Appendix 1, the optimal order quantity Q^* can be found at the Q value where the expected cost function is flat. This is where the expected costs for Q and $Q+1$ units are approximately equal (i.e., $ECost(Q)\approx ECost(Q+1)$). Therefore, Q^* is the smallest value of Q such that the following relationship holds true:

$$P(Q^*)=\sum_{D=0}^{Q^*}p(D)\geq\frac{c_u}{c_u+c_o} \tag{3}$$

Appendix 1 derives equation (3). The value $R=c_u/(c_u+c_o)$ is called the "critical ratio" or "critical fractile" and is always between zero and one. The optimal Q is denoted as Q^* and can be found with a simple search procedure starting at Q = 1 and increasing Q until the above

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[1] A discrete random variable only takes on integer (whole number) values. This could be based on the Poisson distribution, another theoretical discrete distribution, or an empirical discrete distribution (using historical data).

[2] A mathematically concise expression is $C(Q,D)=c_o(Q-D)^+ + c_u(D-Q)^+$, where $(x)^+=\max(x,0)$.

[3] The probability mass function p(D) is the probability that demand is exactly the integer D. The cumulative distribution function P(D) is the probability that demand is less than or equal to D.

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relationship is satisfied. When $c_u = c_o$, the critical ratio is $R = 0.5$, which is consistent with the intuition that suggests that Q^* should be equal to the median demand when the costs are equal.

Newsvendor example with the Poisson distribution

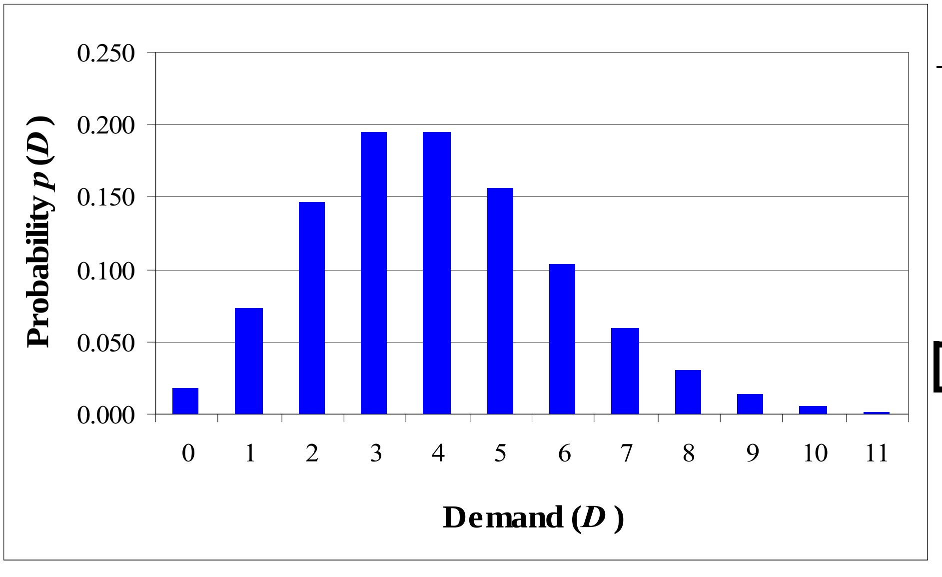
For example, a buyer for a manufacturer must decide how much to make with the last manufacturing run before a product is discontinued. The firm currently has zero in stock and the forecast for the lifetime demand is $\lambda = 4$ units. (The forecast is the mean of the distribution.) The demand over the lifetime of the product is assumed to be a Poisson distributed random variable (a reasonable assumption). The underage cost (c_u) and overage cost (c_o) are estimated to be \$1000 and \$100 per unit, respectively. The c_u parameter is large because a stockout will disappoint customers and because the product will not be manufactured again. The critical ratio is $R = c_u / (c_u + c_o)$ = 1000/1100 = 0.909. Hill, Giard, and Mabert (1989) developed a decision support system to help managers solve the newsvendor problem in this business context.

Figure 1 shows the Poisson probabilities $p(D)$ and the cumulative Poisson probabilities $P(D)$. The optimal (maximum expected profit) value of Q can be found by finding the smallest value of Q such that $P(Q) \geq 0.909$. The optimal value of Q for this problem, therefore, is $Q^* = 7$.

Implementing the model in Excel

The cumulative Poisson distribution can be implemented in Excel with the function POISSON(Q, λ, TRUE). While Excel does not provide a function for the inverse of the cumulative Poisson, it is easy to find the Q that satisfies equation (3) with a simple search. Appendix 4 implements a simple VBA function for Excel for the Poisson inverse Cumulative Distribution Function (CDF) where Q^* = poisson_inverse(R, λ).

Figure 1. Poisson probabilities with mean λ = 4



D	*p*(*D*)	*P*(*D*)	
0	0.018	0.018	
1	0.073	0.092	
2	0.147	0.238	
3	0.195	0.433	
4	0.195	0.629	
5	0.156	0.785	
6	0.104	0.889	
7	0.060	0.949	← Q^*
8	0.030	0.979	
9	0.013	0.992	
10	0.005	0.997	
11	0.002	0.999	

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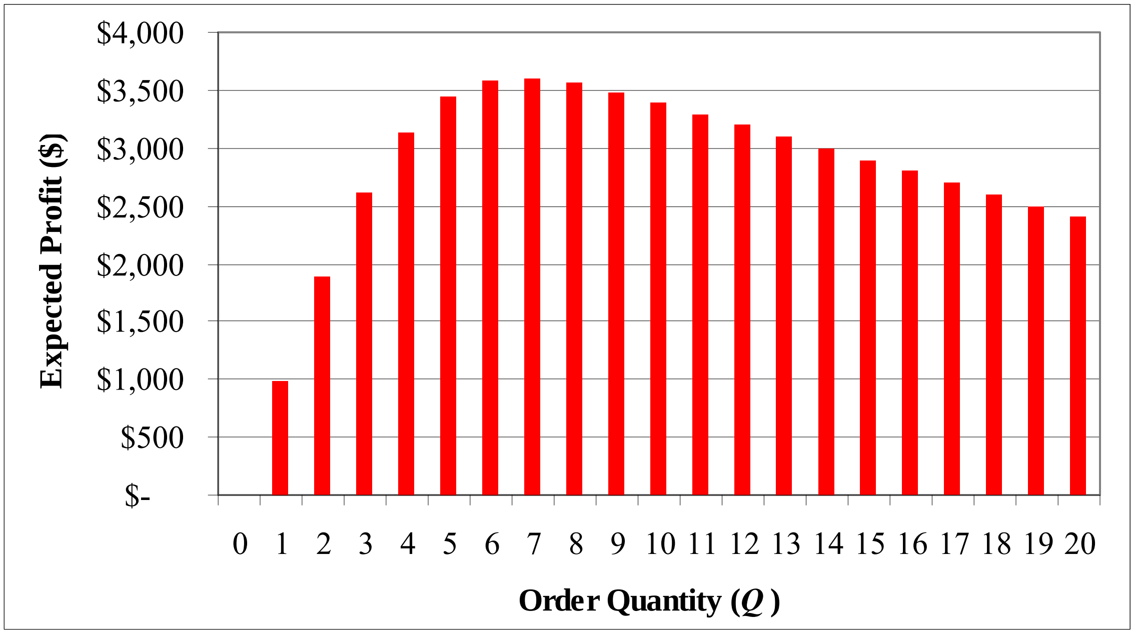
Figure 2 shows the expected profit for this example, again showing the optimal value at $Q^* = 7$ units. Notice that the expected profit does not change very much with small deviations from Q^* and that it is better to err on the high side than on the low side for this example. As derived in Appendix 3, the expected profit is $(p-s+g)(\lambda P(Q-1)-QP(Q))+(p-c+g)Q-g\lambda$, which is \$3,607 for this example.

Figure 2. Expected profit versus order quantity for the example with $Q^* = 7$



3. The newsvendor model with continuous demand

The model

As with the discrete demand case, the cost for order quantity Q and specific demand D is:

$$Cost(Q,D)=\begin{cases} c_o(Q-D) & \text{if } D<Q \\ c_u(D-Q) & \text{if } D\geq Q \end{cases} \tag{4}$$

We assume that demand (D) is a continuous random variable[4] with density function $f(D)$ and cumulative distribution function $F(D)$. The expected cost function is given by:

$$ECost(Q)=\int_{D=0}^{\infty} Cost(D,Q)f(D)dD = c_o\int_{D=0}^{Q}(Q-D)f(D)dD + c_u\int_{D=Q}^{\infty}(D-Q)f(D)dD \tag{5}$$

4 *A continuous random variable can take on any real value, including fractional values.*

This equation is analogous to equation (2) for the discrete demand problem. In order to find the optimal Q, we take the derivative of the expected cost function and set it to zero to find:

$$\frac{dECost(Q)}{dQ} = c_o F(Q) - c_u(1 - F(Q)) = 0$$

$$\Rightarrow F(Q) = \frac{c_u}{c_u + c_o} \qquad (6)$$

$$\therefore Q^* = F^{-1}\left(\frac{c_u}{c_u + c_o}\right)$$

Testing the second derivative proves that Q^* is a global optimum.

As mentioned before, $R = c_u / (c_u + c_o)$ is critical ratio and is always between zero and one. In order to find Q^*, the optimal value of Q, it is necessary to find the Q associated with the cumulative probability distribution so that $F(Q^*) = c_u / (c_u + c_o)$. Mathematicians write this as $Q^* = F^{-1}(c_u / (c_u + c_o))$, where $F^{-1}(.)$ is the inverse of the cumulative distribution function (also called the inverse distribution function).[5] Appendix 2 presents the derivation for equation (6). Appendix 3 derives expressions for the expectations for the number of units sold, lost sales, units salvaged, cost, and profit. This appendix also shows the relationship between the expected profit and expected cost.

Implementing the continuous demand model with the normal distribution

Microsoft Excel includes the inverses for several cumulative distributions, including the normal, lognormal, and gamma distributions. For the normal distribution, the Excel function for the optimal Q^* is NORMINV(R, μ, σ). For example, a newsvendor problem has costs c_o = \$100 and c_u = \$1000 and a critical ratio of $R \approx 0.909$. The demand is normally distributed with $\mu = 4$ and $\sigma = 1$ units. The optimal order quantity is then $Q^* \approx$ NORMINV(0.909, 4, 1) ≈ 5.34 units. The ***Encyclopedia of Operations Management*** (Hill, 2012) includes Excel functions for the inverse cumulative distributions for all commonly used continuous probability distributions.

Implementing the continuous demand model with the triangular distribution

When little or no historical information about demand is available and/or the demand distribution is not symmetrical, the triangular distribution is a practical approach. An experienced person (or team) estimates three parameters – the minimum demand ($D_{\min}$), most likely demand (D_{ml}), and maximum demand ($D_{\max}$). It is best to start with $D_{\min}$ and $D_{\max}$ so that people do not "anchor" on the mode. Excel does not include the triangular distribution or its

[5] *Mathematicians also write this as* $Q^* = \arg\min_Q (ECost(Q))$.

inverse, but the inverse for the triangular distribution is easy to derive and implement (Hill & Sawaya 2004). With the triangular distribution, the optimal order quantity for critical ratio $R = c_u / (c_u + c_o)$ is:

$$Q^* = F^{-1}(R) = \begin{cases} D_{\min} & \text{for } R = 0 \\ D_{\min} + \sqrt{R(D_{\max} - D_{\min})(D_{\text{ml}} - D_{\min})} & \text{for } 0 < R \le (D_{\text{ml}} - D_{\min}) / (D_{\max} - D_{\min}) \\ D_{\max} - \sqrt{(1-R)(D_{\max} - D_{\min})(D_{\max} - D_{ml})} & \text{for } (D_{\text{ml}} - D_{\min}) / (D_{\max} - D_{\min}) < R < 1 \\ D_{\max} & \text{for } R = 1 \end{cases} \quad (7)$$

Appendix 5 presents the VBA code for this function.

4. Newsvendor example with continuous demand

The problem

A retailing firm buys swimsuits for the summer season. The firm buys its swimsuits from a low cost provider in Asia, but is only able to make a single purchase per year. The estimated demand is 5000 units, with a minimum of 2000 and a maximum of 8000 units. The selling price is p = \$20 per unit. The firm pays c = \$5.00 per unit. The firm can sell excess inventory outside North America for a salvage value of s = \$2.00 per unit. The management believes that no significant goodwill is lost with a lost sale (i.e., g = 0). Therefore, the underage cost (cost of a lost sale) is $c_u = p - c + g$ = \$15 and the overage cost (cost of one unit of extra inventory) is $c_o = c - s$ = \$3. The critical ratio is then $R = c_u / (c_u + c_o) = 15/18 \approx 83.3\%$.

The solution with the triangular demand distribution approach

With the triangular distribution, we have ($D_{\min}, D_{\text{ml}}, D_{\max}$) = (2000, 5000, 8000). Using the inverse cumulative distribution function for the triangular distribution (equation (7)), the optimal order quantity is $Q^* \approx F^{-1}(0.833) \approx 6{,}268$ units and the optimal expected profit is approximately \$69,464. Figures 3 and 4 show the demand and the expected profit graphs with the triangular distribution for demand.

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Figure 3. Demand distribution for the example problem with the triangular distribution

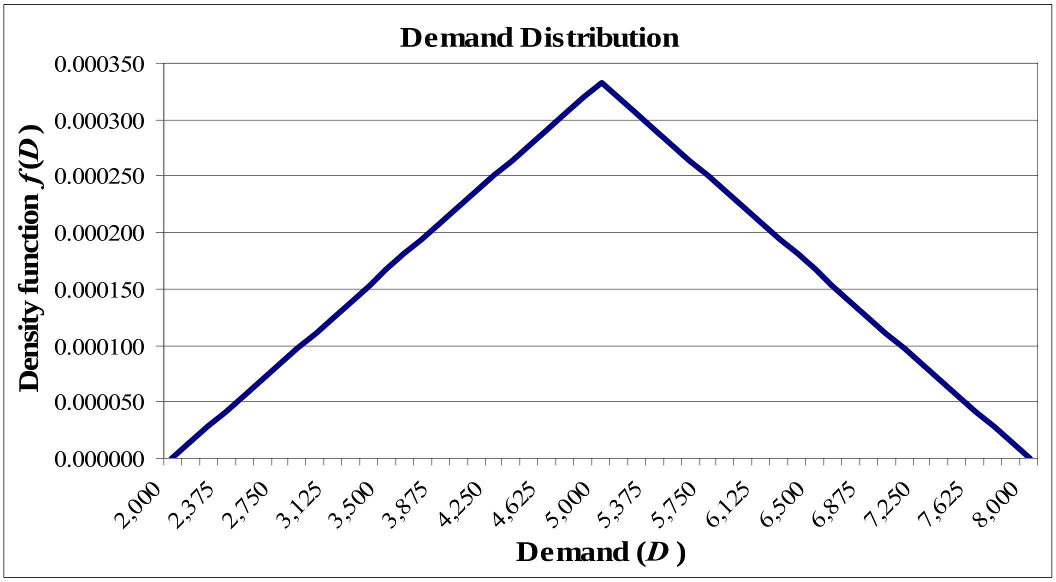
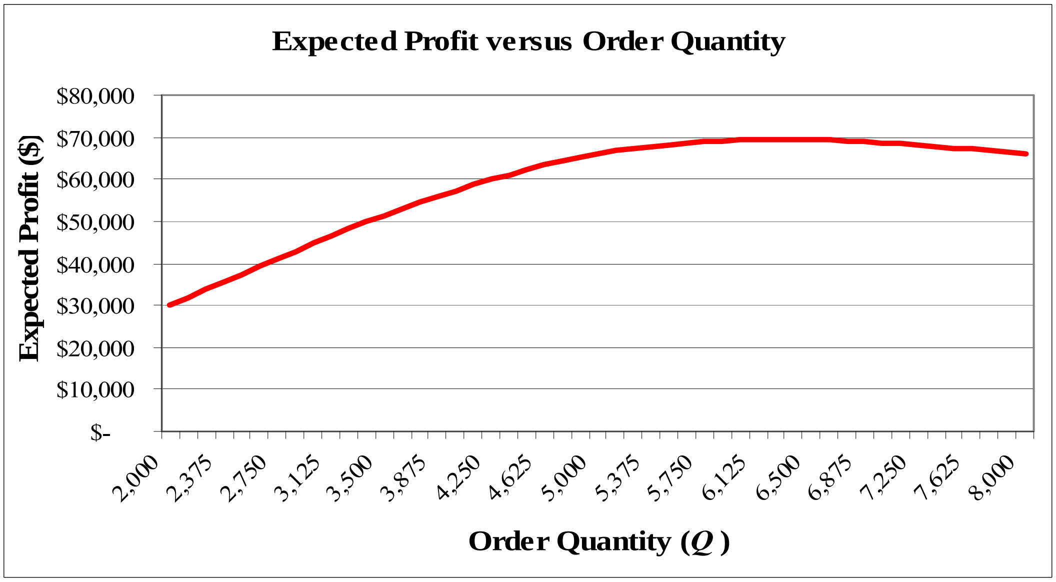


Figure 4. Expected profit for the example problem with the triangular distribution



The solution with the normally distributed demand approach

With the triangular distribution, the estimated standard deviation is $\hat{\sigma} = (D_{\max} - D_{\min})/6$ = (8000 − 2000)/6 = 1000 units. Using the inverse cumulative normal distribution, the optimal order quantity is $Q^* \approx F^{-1}(0.833) \approx 5967$ units and the optimal expected profit is about $70,503. In Excel Q^* can be found with NORMINV(0.833, 5000, 1000). Figures 5 and 6 show the graphs.

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Figure 5. Demand distribution for the example problem with the normal distribution

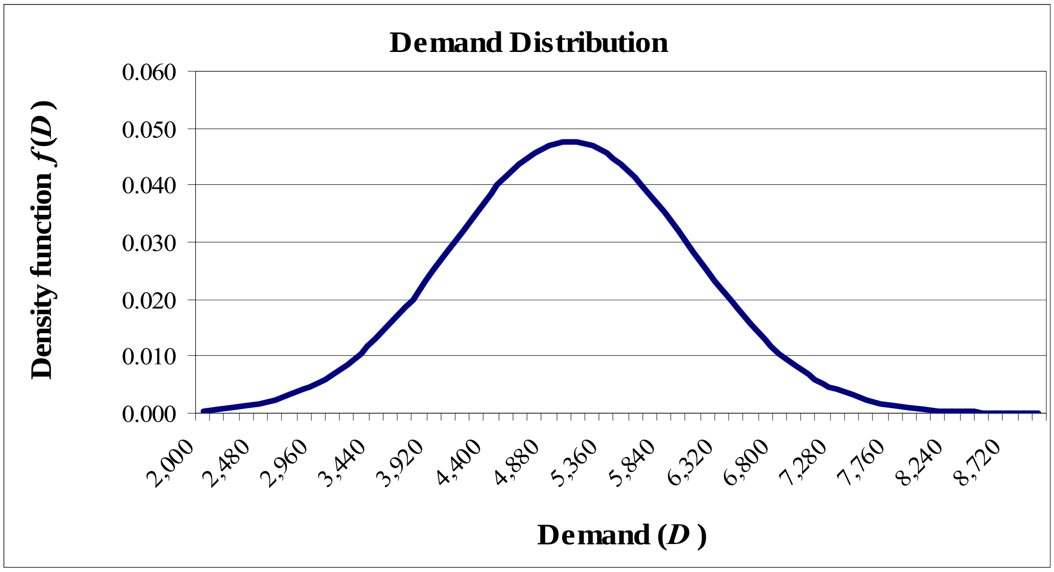
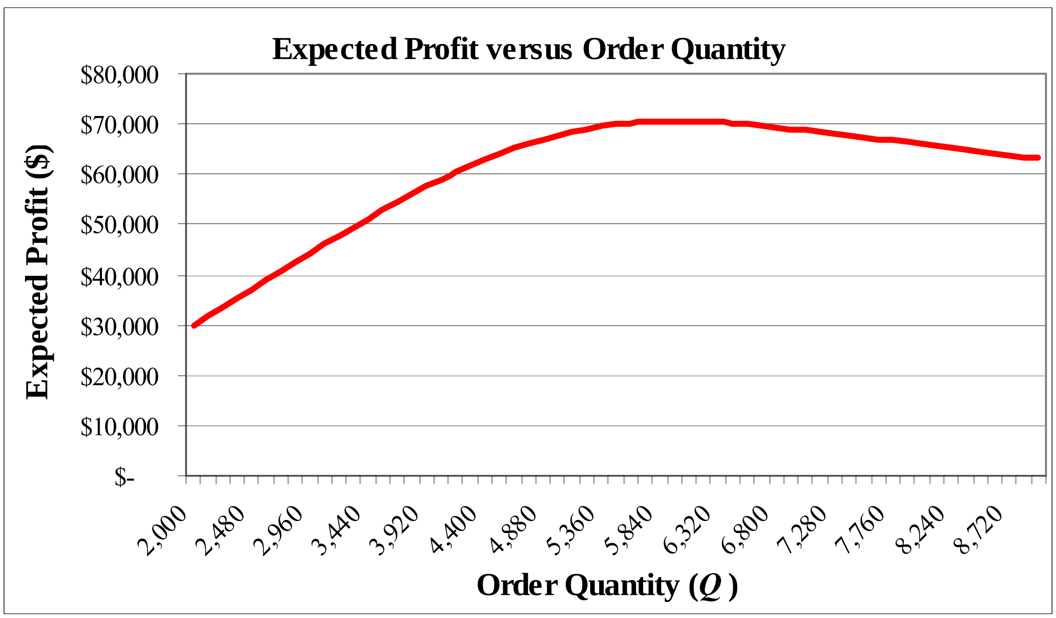


Figure 6. Expected profit for the example problem with the normal distribution



In this example, the triangular and normal distribution approaches have practically the same optimal order quantity and optimal expected profit. However, when the most likely value is not close to the midpoint of the minimum and maximum values (i.e., the distribution is skewed), the optimal solutions may be far apart. When this is true, the triangular distribution approach will likely be a better method than the normal distribution approach.

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5. Estimating the critical ratio

The critical ratio is normally calculated using the equation $R = c_u / (c_u + c_o)$, which requires estimates of both the underage and overage costs. As mentioned in the introduction, in the retail context, the underage cost is price minus unit cost plus lost goodwill (i.e., $c_u = p - c + g$) and the overage cost is unit cost less salvage value (i.e., $c_o = c - s$). While price, cost, and salvage are usually fairly easy to estimate, the lost goodwill (g) is often difficult to estimate, which means that the underage cost and the critical ratio are also hard to estimate.

Goodwill is hard to estimate because it is difficult to predict how customers react to a stockout situation. Some disappointed customers might be satisfied with an alternative product from the retailer or might return at a later date, which means that g is close to zero. However, some customers might leave the store disappointed and never come back, which means that the retailer would lose the net present value of the lifetime stream of profit from those customers. Even worse, some customers might give a bad report to many others, which might further damage the retailer's brand, sales, and profits. Goodwill is also hard to estimate in other newsvendor problem contexts such as the overbooking context where it is difficult to estimate the lost goodwill associated with turning away a passenger at the boarding gate for a departing flight.

Another approach for calculating the critical ratio estimates the ratio of the cost parameters $r = c_u / c_o$ without estimates of either the underage or overage cost. It is easy to show that once r is known, the critical ratio can be calculated as $R = r / (r+1)$.[6] When $r = c_u / c_o = 1$, the critical ratio for the newsvendor problem is R = 1/(1+1) = 0.50, which is the median of the demand distribution. When r is much greater than one, the critical ratio R is close to 1, which leads to an optimal Q on the far right tail of the demand distribution. When r is close to zero, the critical ratio R is close to zero, which leads to an optimal Q on the far left tail of the demand distribution.

For example, it might be difficult to estimate goodwill parameter g and the associated underage cost c_u, but the decision makers might be able to estimate that the cost of a lost sale is ten times higher than the cost of having too much inventory at the end of the period. For this situation, the ratio $r = c_u / c_o$ is 10 and the critical ratio is $R = r / (r+1)$ = 10/(10+1) = 10/11 ≈ 0.909.

6. Behavioral issues with the newsvendor problem

Reward systems

In this author's experience, decision makers (buyers, analysts, managers) who solve the newsvendor problem as a part of their job often make bad decisions. In this author's opinion, the

[6] *Proof:* $R = \frac{c_u}{c_u + c_o} = \frac{c_u}{c_u + c_o} \cdot \frac{(1/c_o)}{(1/c_o)} = \frac{c_u / c_o}{c_u / c_o + 1} = \frac{r}{r+1}$, *where* $r = c_u / c_o$.

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main issue is that reward systems are not aligned with the economics. In other words, the decision makers are instructed to optimize expected profit, but are then also told to reduce lost sales or reduce excess inventory. These mixed messages lead the decision makers to respond to the voice that is "yelling" the loudest at the moment and to ignore (or at least discount) the hard to measure economics.

For example, in a project this author did with a large music retailer, the cost of excess inventory for new releases was low because the retailer could return CDs to the manufacturer for a small restocking fee (about 15% of the cost). The cost of a stockout for the retailer was high due to high margins (about $10 per CD). Applying the newsvendor model, the retailer's buyers should have been aggressively overbuying on a consistent basis. However, excess inventory was easy to measure and lost sales were hard to measure, which often led buyers to give more weight to excess inventory and less weight to lost sales in their buying decisions. In other words, it appeared that buyers were being driven by their reward system to under-buy even though the economics should have led them to overbuy. This meant that the critical ratio for the buyers was different from the critical ratio for the retailer, which resulted in a "misalignment" between the goals for the buyers and the retail firm. The buyers' overage cost was $c_o = c - s + b$, where b is the buyer's reputational "cost" per unit remaining at the end of the selling season. The b and g parameters are both difficult to estimate, but can sometimes be imputed (inferred) from historical buyer behavior (Olivares, Terwiesch, & Cassorla, 2008).[7]

Low margin (critical ratio) conditions

Several research papers have used human experiments to study the behavioral issues with the newsvendor problem (e.g., Schweitzer & Cachon 2000; Bolton and Katok 2008). Nearly all of this research has found that subjects tend to under-buy in high critical ratio (high margin) situations and overbuy in low critical ratio (low margin) situations. This pattern cannot be explained by risk aversion, risk-seeking preferences, loss avoidance, waste aversion, or understanding opportunity costs. Moritz and Hill (2010) found that subjects have "cognitive dissonance" in low margin situations because they are faced with the dilemma of meeting customer demand (which suggests a large order quantity) and optimizing expected profit (which suggests a small order quantity). They conclude that subjects tend to estimate a positive goodwill parameter (e.g., $g > 0$), even when told that goodwill is not lost with a shortage (e.g., $g = 0$).

High margin (critical ratio) conditions

Moritz, Hill, and Donohue (2010) found that decision makers in a high margin condition tended to anchor on the previous period demand when making newsvendor decisions over several periods. They also found that the simple three-question cognitive reflection test (CRT) developed by Frederick (2005) predicted the degree to which decision makers anchored on the previous period demand, where high CRT people had significantly less anchoring. CRT

[7] *Do not confuse the buyer's reputational cost parameter (b), which is for overbuying, and the retailer's lost goodwill parameter (g), which is for underbuying.*

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measures the degree to which people allow their system 1 (automatic, impulsive) thinking to be moderated by their system 2 (analytical) thinking. In other words, the CRT measures the degree to which individuals are more patient and less impulsive when presented with a judgmental task.

7. Conclusions

The newsvendor logic is fundamental to solving many operations problems. The newsvendor model provides both useful intuition and a useful tool. Explicitly defining the overbuying and under-buying costs, and calculating the critical ratio can often lead to better economic decisions than those made only on the basis of experience, intuition, myopic reward systems, politics, power, or personalities.

If managers are willing and able to make some assumptions about the form of the demand distribution and estimate the demand distribution and cost parameters, they can imbed the newsvendor model in a decision support system to help buyers make better economic decisions. This type of decision support system is particularly valuable for retail "style" goods where many decisions have to be made routinely every day and where these decisions have a significant financial impact on the firm.

The inputs to newsvendor model include (a) the form of the demand distribution (e.g., normal,), (b) the parameters of the demand distribution (e.g., mean), and (c) estimates of the overage and underage cost parameters. The goodwill parameter, a component of the underage cost, is often difficult to estimate. This paper introduced the concept of the buyer's reputational "cost" per unit remaining at the end of the selling season. This parameter is also difficult to estimate.

Decision makers (buyers, analysts, managers) who solve the newsvendor problem as a part of their job often make bad decisions. The reward systems should be designed to align buyer behavior with the firm's economic objectives. Also, it appears that in low margin situations, buyers tend to be biased towards inferring a higher goodwill parameter and in high margin situations, buyers with low CRT tend to put too much weight on last season's actual demand.

The newsvendor economic logic appears in many different business contexts such as buying for a one-time selling season, making a final production run, setting safety stocks, setting target inventory levels, and making capacity decisions. These contexts all have a single decision variable, random demand, and known overage and underage costs. The newsvendor model provides a useful tool for solving these problems and practical insights into how to think about these problems.

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Appendix 1: Derivation of the newsvendor model with discrete demand[8]

The optimal expected cost will be where the expected costs for ordering Q units is approximately the same as the expected cost for ordering Q + 1 units. This is the point at the bottom of the total expected cost curve where the curve is flat, which is the global minimum expected cost. Setting $ECost(Q) = ECost(Q+1)$ and applying equation (2), we find:

$$c_o \sum_{D=0}^{Q-1} p(D)(Q-D) + c_u \sum_{D=Q}^{\infty} p(D)(D-Q) = c_o \sum_{D=0}^{Q} p(D)(Q+1-D) + c_u \sum_{D=Q+1}^{\infty} p(D)(D-Q-1) \quad (8)$$

However, the left side can be rewritten as $c_o \sum_{D=0}^{Q} p(D)(Q-D) + c_u \sum_{D=Q+1}^{\infty} p(D)(D-Q)$ because $D-Q=0$ when $Q=D$. Combining terms and defining the cumulative distribution function as $F(Q) = \sum_{D=0}^{Q} p(D)$ yields:

$$\begin{aligned} c_o \sum_{D=0}^{Q} p(D) - c_u \sum_{D=Q+1}^{\infty} p(D) &= 0 \\ c_o F(Q) - c_u (1 - F(Q)) &= 0 \\ \therefore F(Q) = \sum_{D=0}^{Q} p(D) &= \frac{c_u}{c_u + c_o} \end{aligned} \quad (9)$$

Therefore, the optimal order quantity for the discrete demand newsvendor problem can be found by finding the smallest Q such that $F(Q) = \sum_{D=0}^{Q} p(D) \geq c_u / (c_u + c_o)$, where the quantity $R = c_u / (c_u + c_o)$ is the critical ratio (fractile). Note that this result requires no assumptions about the demand distribution other than it must be a discrete distribution. The next section proves that expression (9) also holds for continuous demand distributions.

Appendix 2: Derivation of the newsvendor model for continuous demand

For order quantity Q and specific demand D, the cost is:

$$Cost(Q,D) = \begin{cases} c_o (Q-D) & \text{for } D \leq Q \\ c_u (D-Q) & \text{for } D > Q \end{cases} \quad (10)$$

For continuous demand with density $f(D)$, the expected cost is:

[8] *The derivations in the appendices show many more intermediate mathematical steps than are normally shown in a research paper. This is done to help all readers understand the details of the derivations.*

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$$\begin{aligned}
ECost(Q) &= \int_{D=0}^{\infty} Cost(D,Q) f(D)dD \\
&= c_o \int_{D=0}^{Q} (Q-D) f(D)dD + c_u \int_{D=Q}^{\infty} (D-Q) f(D)dD \\
&= c_o Q \int_{D=0}^{Q} f(D)dD - c_o \int_{D=0}^{Q} Df(D)dD + c_u \int_{D=Q}^{\infty} Df(D)dD - c_u Q \int_{D=Q}^{\infty} f(D)dD \\
&= c_o QF(Q) - c_o H(Q) + c_u(\mu - H(Q)) - c_u Q(1-F(Q)) \\
&= c_o QF(Q) + c_u QF(Q) - c_o H(Q) - c_u H(Q) + c_u \mu - c_u Q \\
&= (c_u + c_o)(QF(Q) - H(Q)) + c_u(\mu - Q)
\end{aligned} \tag{11}$$

where $F(Q)$ is the demand distribution function evaluated at Q and $\int_{D=Q}^{\infty} Df(D)dD = \mu - H(Q)$.

Note: In equation (11), the mean demand is $\mu = \int_{D=0}^{\infty} Df(D)dD$ and is called the "complete expectation" because the range of integration is $(0,\infty)$. The partial expectation of demand is $H(Q) = \int_{D=0}^{Q} Df(D)dD$ with a range of integration $(0,Q)$.[9] Given that $\mu = H(\infty)$, it is clear that $\int_{D=Q}^{\infty} Df(D)dD = \mu - H(Q)$.

By the fundamental law of calculus, $F'(Q) = f(Q)$ and according to Leibniz's rule[10] $H'(Q) = Qf(Q)$. Therefore, the first derivative of $ECost(Q)$ with respect to Q is:

$$\begin{aligned}
\frac{dECost(Q)}{dQ} &= (c_u + c_o)(QF'(Q) + F(Q) - H'(Q)) - c_u \\
&= (c_u + c_o)(Qf(Q) + F(Q) - Qf(Q)) - c_u \\
&= (c_u + c_o)F(Q) - c_u
\end{aligned} \tag{12}$$

---

9 *Winkler, Roodman, and Britney (1972) use the term partial moment rather than partial expectation. We assume that demand is always non-negative (i.e., $D \geq 0$). Winkler et al. use the notation $E_0^Q(D)$ for the partial expectation of the random variable D in the range (0,Q). This paper will use the simpler notation $H(Q)$.*

10 *Leibniz's rule states that* $\frac{d}{dy}\int_{x=g(y)}^{x=h(y)} r(x,y)dx = \int_{x=g(y)}^{x=h(y)} \frac{\partial r(x,y)}{\partial y}dx + r(h(y),y)\frac{dh(y)}{dy} - r(g(y),y)\frac{dg(y)}{dy}$. *In this situation,* $H'(Q) = dH(Q)/dQ = \frac{d}{dQ}\int_{D=0}^{Q} Df(D)dD = Qf(Q)$, where $y = Q$, $x = D$, $h(Q) = Q$, $g(Q) = 0$, *and* $r(x,y) = r(D,Q) = Df(D)$.

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Setting this derivative to zero leads to:

$$F(Q) = \frac{c_u}{c_u + c_o} \tag{13}$$

where the quantity $R = c_u / (c_u + c_o)$ is the critical ratio (fractile). The second derivative of $ECost(Q)$ is $d^2 ECost(Q)/dQ^2 = (c_o + c_u) f(Q)$, which is non-negative for all values of Q. Therefore, $ECost(Q)$ is a convex function and $Q^* = F^{-1}(c_u / (c_u + c_o))$ is the globally optimal order quantity. Note that this result does not require any assumptions about the demand distribution and therefore is true for all continuous probability distributions.

Appendix 3: Derivations of expected values

The derivations in this section are developed for continuous demand. Expectations for discrete demand are identical when integrals are replaced by summations, $f(D)$ is replaced by $p(D)$. These derivations show the details to make the mathematical reasoning as accessible as possible for the non-mathematical reader.

Expected number of units sold

For order quantity Q and specific demand D, the number of units sold is:

$$Sold(Q,D) = \begin{cases} D & \text{for } D \le Q \\ Q & \text{for } D > Q \end{cases} \tag{14}$$

An alternative expression for the number of units sold is $\min(D,Q)$. For continuous demand with density $f(D)$, the expected number of units sold is then given by:

$$\begin{aligned} ESold(Q) &= \int_{D=0}^{\infty} Sold(Q,D) f(D) dD \\ &= \int_{D=0}^{Q} D f(D) dD + Q \int_{D=Q}^{\infty} f(D) dD \\ &= H(Q) + Q(1 - F(Q)) \end{aligned} \tag{15}$$

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where $H(Q)$ is the partial expectation of demand, which is defined as $H(Q) = \int_{D=0}^{Q} Df(D)dD$, and $\int_{D=Q}^{\infty} f(D)dD = 1 - F(Q)$. Winkler, Roodman, and Britney (1972) prove that the partial expectation (partial first moment) for a normally distributed random variable is:

$$H(Q) = \int_{D=0}^{Q} Df(D)dD = \mu F_u(z) - \sigma f_u(z) \tag{16}$$

where μ and σ are the mean and standard deviation of demand, $F_u(z)$ and $f_u(z)$ are the cumulative and probability density functions for the standard normal distribution, and $z = (Q-\mu)/\sigma$. Note that $F(Q) = F_u(z)$, but that $f(Q) \neq f_u(z)$. Therefore, for normally distributed demand, the expected number of units sold is:

$$\begin{aligned} ESold(Q) &= H(Q) + Q(1 - F(Q)) \\ &= \mu F_u(z) - \sigma f_u(z) + Q(1 - F(Q)) \\ &= \mu F_u(z) - \sigma f_u(z) + Q - QF(Q) \\ &= \mu F(Q) - \sigma f_u(z) + Q - QF(Q) \\ &= Q + (\mu - Q)F(Q) - \sigma f_u(z) \end{aligned} \tag{17}$$

Expected number of units of lost sales

For order quantity *Q* and specific demand *D*, the number of units of lost sales is:

$$Lost(Q,D) = \begin{cases} 0 & \text{for } D \le Q \\ D - Q & \text{for } D > Q \end{cases} \tag{18}$$

Alternative expressions for lost sales include $\max(D-Q,0)$ and $(D-Q,0)^+$. For continuous demand with density function $f(D)$, the expected units of lost sales is:

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$$\begin{aligned} ELost(Q) &= \int_{D=0}^{\infty} Lost(Q,D) f(D) dD \\ &= \int_{D=Q}^{\infty} (D-Q) f(D) dD \\ &= \int_{D=Q}^{\infty} Df(D) dD - Q \int_{D=Q}^{\infty} f(D) dD \\ &= \mu - H(Q) - Q(1-F(Q)) \end{aligned} \tag{19}$$

where $\mu = \int_{D=-\infty}^{Q} Df(D)dD + \int_{D=Q}^{\infty} Df(D)dD$ and $\int_{D=Q}^{\infty} f(D)dD = 1 - F(Q)$.

For normally distributed demand, $H(Q)$ can be replaced with equation (16) to find the expected number of units of lost sales:

$$\begin{aligned} ELost(Q) &= \mu - H(Q) - Q(1-F(Q)) \\ &= \mu - \mu F_u(z) + \sigma f_u(z) - Q(1-F(Q)) \\ &= \mu(1-F(Q)) - Q(1-F(Q)) + \sigma f_u(z) \\ &= (\mu - Q)(1-F(Q)) + \sigma f_u(z) \end{aligned} \tag{20}$$

Expected number of units salvaged

For order quantity *Q* and specific demand *D*, the number of units salvaged is:

$$Salvage(Q,D) = \begin{cases} Q - D & \text{for } D \le Q \\ 0 & \text{for } D > Q \end{cases} \tag{21}$$

Alternative expressions for the number of units salvaged include $\max(Q-D,0)$ and $(Q-D,0)^+$. For continuous demand with density function $f(D)$, the expected number of units salvaged is:

$$\begin{aligned} ESalvage(Q) &= \int_{D=0}^{\infty} Salvage(Q,D) f(D) dD \\ &= \int_{D=0}^{Q} (Q-D) f(D) dD \\ &= Q\int_{D=0}^{Q} f(D) dD - \int_{D=0}^{Q} Df(D) dD \\ &= QF(Q) - H(Q) \end{aligned} \tag{22}$$

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For normally distributed demand, $E(Q)$ can be replaced with equation (16) to find the expected number of units salvaged:

$$\begin{aligned} ESalvage(Q) &= QF(Q) - H(Q) \\ &= QF(Q) - \mu F_u(z) + \sigma f_u(z) \\ &= QF(Q) - \mu F(Q) + \sigma f_u(z) \\ &= (Q-\mu)F(Q) + \sigma f_u(z) \end{aligned} \tag{23}$$

Expected cost

As developed in equation (11), the expected cost for continuous demand is:

$$ECost(Q) = (c_u + c_o)(QF(Q) - H(Q)) + c_u(\mu - Q) \tag{24}$$

For normally distributed demand, $E(Q)$ can be replaced with equation (16) to find the expected cost:

$$\begin{aligned} ECost(Q) &= (c_u + c_o)(QF(Q) - \mu F_u(z) + \sigma f_u(z)) + c_u(\mu - Q) \\ &= (c_u + c_o)((Q-\mu)F(Q) + \sigma f_u(z)) + c_u(\mu - Q) \end{aligned} \tag{25}$$

where $z = (Q-\mu)/\sigma$ and $F_u(z)$ and $f_u(z)$ are the CDF and PDF for the standard normal. (Note that $F_u(z) = F(Q)$ and $f_u(z) \neq f(Q)$.)

Expected profit

For order quantity *Q* and specific demand *D*, the profit is:

$$Profit(Q,D) = \begin{cases} pD + s(Q-D) - cQ & \text{if } D \leq Q \\ pQ - g(D-Q) - cQ & \text{if } Q < D \end{cases} \tag{26}$$

The expected profit function, therefore, is:

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$$EProfit(Q) = \int_{D=0}^{\infty} Profit(Q,D) f(D) dD$$
$$= \int_{D=0}^{Q} [pD + s(Q-D)] f(D) dD + \int_{D=Q}^{\infty} [pQ - g(D-Q)] f(D) dD - cQ$$
$$= p \int_{D=0}^{Q} Df(D) dD + sQ \int_{D=0}^{Q} f(D) dD - s \int_{D=0}^{Q} Df(D) dD$$
$$+ pQ \int_{D=Q}^{\infty} f(D) dD - g \int_{D=Q}^{\infty} Df(D) dD + gQ \int_{D=Q}^{\infty} f(D) dD - cQ$$
$$= pH(Q) + sQF(Q) - sH(Q) + pQ(1-F(Q)) - g(\mu - H(Q)) + gQ(1-F(Q)) - cQ \qquad (27)$$
$$= pH(Q) + sQF(Q) - sH(Q) + pQ - pQF(Q) - g\mu + gH(Q) + gQ - gQF(Q) - cQ$$
$$= pH(Q) - sH(Q) + gH(Q) - pQF(Q) + sQF(Q) - gQF(Q) + pQ - cQ + gQ - g\mu$$
$$= (p-s+g)H(Q) - (p-s+g)QF(Q) + (p-c+g)Q - g\mu$$
$$= (p-s+g)(H(Q) - QF(Q)) + (p-c+g)Q - g\mu$$

For normally distributed demand, $H(Q)$ can be replaced with equation (16) to find the expected profit:

$$EProfit(Q) = (p-s+g)(H(Q) - QF(Q)) + (p-c+g)Q - g\mu$$
$$= (p-s+g)(\mu F(Q) - \sigma f_u(z) - QF(Q)) + (p-c+g)Q - g\mu \qquad (28)$$
$$= (p-s+g)((\mu - Q)F(Q) - \sigma f_u(z)) + (p-c+g)Q - g\mu$$

Relationships between expected values

All units ordered at the beginning of the period must be either sold or salvaged at the end of the period:

$$Q = Sold(D,Q) + Salvage(D,Q) \qquad (29)$$

Taking expectations of both sides, leads to the relationship:

$$Q = ESold(Q) + ESalvage(Q) \qquad (30)$$

The demand in a period must be converted into either a sale or a lost sale. In other words, the demand is always the sum of the number of units sold and the number of units of lost sales. For any order quantity and demand realization:

$$D = Sold(D,Q) + Lost(D,Q) \qquad (31)$$

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Taking expectations of both sides, it is clear that the average demand is the sum of the expected number of units sold and the expected number of units of lost sales:

$$\mu = ESold(Q) + ELost(Q) \tag{32}$$

The expected cost in equation (24) can be related to expected profit as follows:

$$\begin{aligned} ECost(Q) &= (c_u + c_o)(QF(Q) - H(Q)) + c_u(\mu - Q) \\ &= (p - c + g + c - s)(QF(Q) - H(Q)) + (p - c + g)(\mu - Q) \\ &= (p - s + g)(QF(Q) - H(Q)) + (p - c + g)\mu - (p - c + g)Q \\ &= (p - c + g)\mu - (p - s + g)(H(Q) - QF(Q)) - (p - c + g)Q \\ &= (p - c)\mu + g\mu - (p - s + g)(H(Q) - QF(Q)) - (p - c + g)Q) + g\mu - g\mu \\ &= (p - c)\mu + g\mu - [(p - s + g)(H(Q) - QF(Q)) + (p - c + g)Q - g\mu] - g\mu \\ \therefore ECost(Q) &= (p - c)\mu - EProfit(Q) \\ \Rightarrow EProfit(Q) &= (p - c)\mu - ECost(Q) \end{aligned} \tag{33}$$

In other words, for any order quantity Q, the expected profit is the profit for selling the average demand, $(p - c)\mu$, less the expected cost. The expected cost, therefore, can be interpreted as the cost of demand variability and the organization should be willing to pay that amount to reduce uncertainty to zero. In other words, when the demand has no variability the expected profit is $(p - c)\mu$ and the expected cost is zero. Given that the quantity $(p - c)\mu$ is a constant and is independent of Q, maximizing expected profit is equivalent to minimizing expected cost.

Summary of the expected value equations

Tables 1 and 2 summarize the expectations for all continuous and discrete demand distributions. Table 3 summarizes the relationships between expected values for the newsvendor problem for any demand distribution. Tables 4 and 5 summarize the expectations for normally distributed demand (a continuous demand distribution) and Poisson distributed demand (a discrete demand distribution).

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Table 1. Summary of expected values for all continuous demand distributions[11]

Expected units sold	$ESold(Q) = H(Q) + Q(1 - F(Q))$
Expected salvage	$ESalvage(Q) = QF(Q) - H(Q)$
Expected lost sales	$ELost(Q) = \mu - H(Q) - Q(1 - F(Q))$
Expected cost [12]	$ECost(Q) = (c_u + c_o)(QF(Q) - H(Q)) + c_u(\mu - Q)$
Expected profit	$EProfit(Q) = (p - s + g)(H(Q) - QF(Q)) + (p - c + g)Q - g\mu$ $= p \cdot ESold(Q) + s \cdot ESalvage(Q) - g \cdot ELost(Q) - cQ$

Table 2. Summary of expected values for all discrete demand distributions[13]

Expected units sold	$ESold(Q) = H(Q) + Q(1 - F(Q))$
Expected salvage	$ESalvage(Q) = QF(Q) - H(Q)$
Expected lost sales	$ELost(Q) = \mu - H(Q) - Q(1 - F(Q))$
Expected cost [14]	$ECost(Q) = (c_u + c_o)(QF(Q) - H(Q)) + c_u(\mu - Q)$
Expected profit	$EProfit(Q) = (p - s + g)(H(Q) - QF(Q)) + (p - c + g)Q - g\mu$ $= p \cdot ESold(Q) + s \cdot ESalvage(Q) - g \cdot ELost(Q) - cQ$

Table 3. Relationships between expectations for all demand distributions

All units ordered will always be either sold or salvaged.	$Q = ESold(Q) + ESalvage(Q)$
The realized demand will always be equal to the actual number of units sold plus the lost sales.	$D = Sold(Q, D) + Lost(Q, D)$ $= \min(D, Q) + \max(D - Q, 0)$
Average demand is always the expected units sold plus the expected units of lost sales.	$\mu = ESold(Q) + ELost(Q)$
Expected profit is always the profit for the average demand less the expected cost.	$EProfit(Q) = (p - c)\mu - ECost(Q)$

Table 4. Summary of expected values for normally distributed demand[15]

Expected units sold	$ESold(Q) = Q + (\mu - Q)F(Q) - \sigma f_u(z)$
Expected salvage	$ESalvage(Q) = (Q - \mu)F(Q) + \sigma f_u(z)$
Expected lost sales	$ELost(Q) = (\mu - Q)(1 - F(Q)) + \sigma f_u(z)$

---

[11] *The partial expectation for continuous demand is defined as* $H(Q) = \int_{D=0}^{Q} Df(D)dD$.

[12] *The expected cost is the sum of the expected underage cost plus the expected overage cost.*

[13] *The partial expectation for discrete demand is defined as* $H(Q) = \sum_{D=0}^{Q} Dp(D)$.

[14] *The expected cost is the sum of the expected underage and overage costs.*

[15] *The expected values for the normal distribution are derived by replacing* $H(Q)$ *with* $\mu F_u(z) - \sigma f_u(z)$, *where* $z = (Q - \mu)/\sigma$. *The proof for this relationship can be found in Winkler, Roodman, and Britney (1972).*

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Expected cost	$ECost(Q) = (c_u + c_o)((Q - \mu)F(Q) + \sigma f_u(z)) + c_u(\mu - Q)$
Expected profit	$EProfit(Q) = (p - s + g)((\mu - Q)F(Q) - \sigma f_u(z)) + (p - c + g)Q - g\mu$

Table 5. Summary of expected values for Poisson[16] distributed demand with mean *λ*

Expected units sold	$ESold(Q) = \lambda F(Q-1) + Q(1 - F(Q))$
Expected salvage	$ESalvage(Q) = QF(Q) - \lambda F(Q-1)$
Expected lost sales	$ELost(Q) = \lambda - \lambda F(Q-1) - Q(1 - F(Q))$
Expected cost	$ECost(Q) = (c_u + c_o)(QF(Q) - \lambda F(Q-1)) + c_u(\lambda - Q)$
Expected profit	$EProfit(Q) = (p - s + g)(\lambda F(Q-1) - QF(Q)) + (p - c + g)Q - g\lambda$

The expected values for other distributions can be found using the partial expectation functions presented in Table 6. These equations were derived by the author from the tail conditional expectation functions presented in Landsman and Valdez (2005).

Table 6. Partial expectation functions

Continuous distributions	
Normal	$H(x) = \mu F_u(z) - \sigma f_u(z)$, where $z = (x - \mu)/\sigma$
Lognormal	$H(x) = e^{\mu + \sigma^2/2}(1 - F_{Normal}((\mu + \sigma^2 - \ln(x))/\sigma))$
Exponential	$H(x) = (\mu + x)F_{Exp}(x) - x$
Gamma	$H(x) = \mu F_{Gamma}(x \mid \alpha + 1, \beta)$
Discrete distributions	
Poisson	$H(x) = \mu F_{Poisson}(x-1)$
Binomial	$H(x) = \mu F_{Binomial}(x - 1 \mid p, n-1)$
Negative binomial	$H(x) = \mu F_{NB}(x - 1 \mid p, \alpha + 1)$

Appendix 4: VBA code for the inverse of the triangular CDF

```
Function triangular_inverse(p, a, b, c) As Double
'
'  Compute the inverse of the triangular distribution at probability p
'  given triangularly distributed demand with parameters
'  (minimum, most likely, maximum) = (a, b, c).
'
   If p <= (b - a) / (c - a) Then
      triangular_inverse = a + Sqr(p * (c - a) * (b - a))
   Else
      triangular_inverse = c - Sqr((1 - p) * (c - a) * (c - b))
```

[16] *Hadley and Whitin (1963) prove that the partial expectation for the Poisson distribution is* $H(Q) = \lambda P(Q-1)$.

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```
   End If
End Function
```

Appendix 5: VBA code for the inverse of the Poisson CDF

```
Function poisson_inverse(p, lambda)
'   p =cumulative probability and lambda = mean of the Poisson distribution.
'   This routine truncates the result at xmax = 60.
Dim x As Integer
Const xmax = 60
   For x = 1 To xmax
      poisson_inverse = x
      If Application.WorksheetFunction.Poisson(x, lambda, True) >= p Then Exit Function
   Next x
   MsgBox “poisson_inverse(“ & Format(p, “0.00%”) & “) was truncated at “ _
      & Val(xmax) & “.”, vbExclamation
End Function
```

Related resources from Clamshell Beach Press: The companion Excel workbook “newsvendor model.xls” is available for download from www.ClamshellBeachPress.com. Other related Excel workbooks from Clamshell Beach Press include “slowmove.xls” and “safety stock.xls.” The “Seasonal Buying” paper applies the newsvendor logic to the retail buying context. The “Triangular Distribution” paper presents the details of the triangular distribution.

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